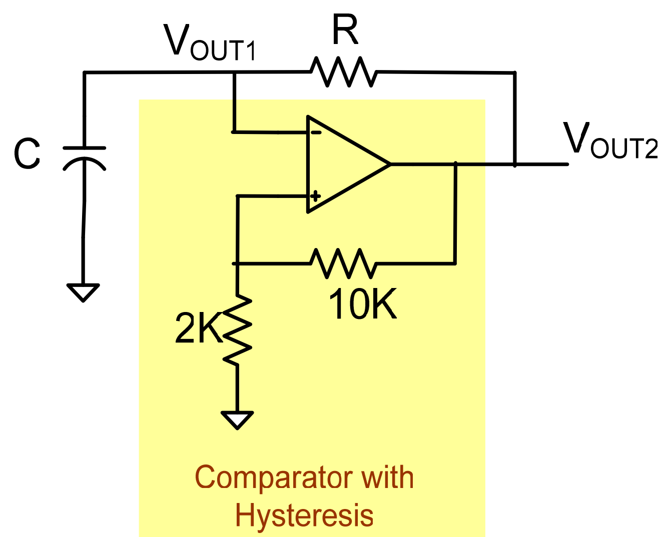
EE 230
Lecture 23

Nonlinear Op Amp Applications

– Waveform Generators

Quiz 17

An oscillator based upon a comparator with hysteresis is shown. If V_{STAH}=12V and V_{SATL}=-12V, determine the peak value of



And the number is ?

1

3

8

5

4

?

2

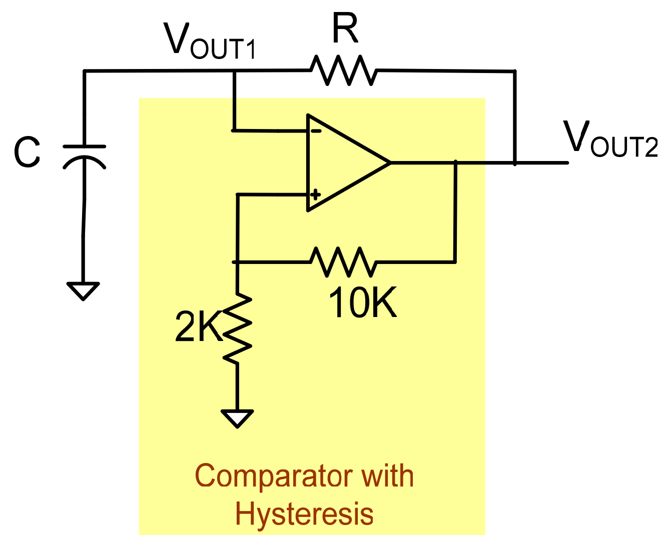
6

9

7

Quiz 17

An oscillator based upon a comparator with hysteresis is shown. If V_{STAH}=12V and V_{SATL}=-12V, determine the peak value of

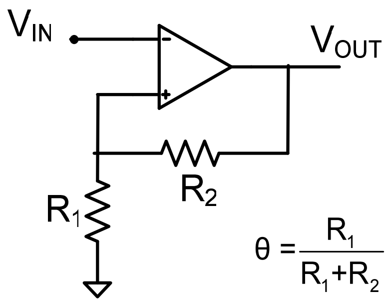


Solution: The peak value of the V_{OUT1} waveform is determined by the boundaries of the Hysteresis window

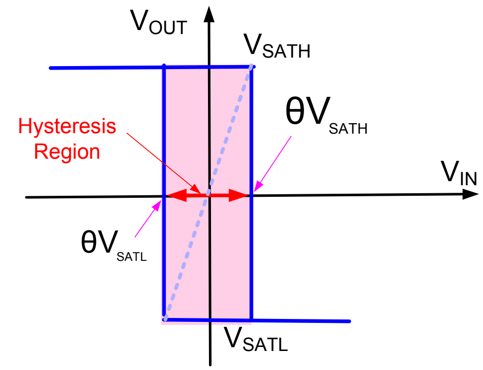
$$V_{OUT1MAX} = V_{SATH}\frac{R_1}{R_1+R_2} = 12V\frac{2K}{12K} = 2V$$

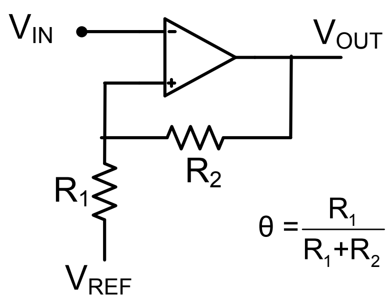
Correction from Last Lecture

Modifications of Comparator with Hysteresis

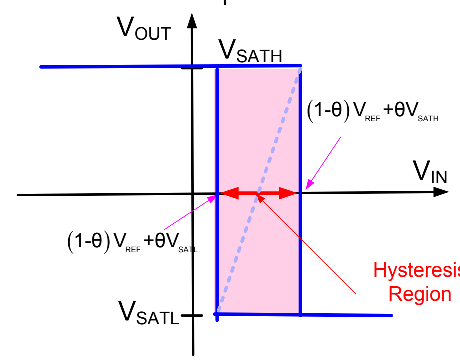


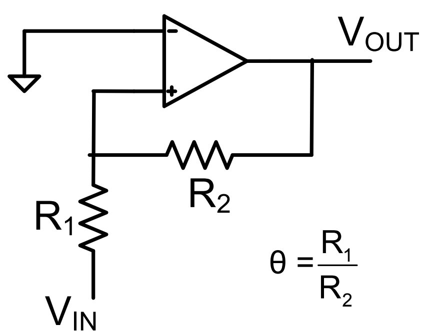
$$\theta = \frac{R_1}{R_1+R_2}$$





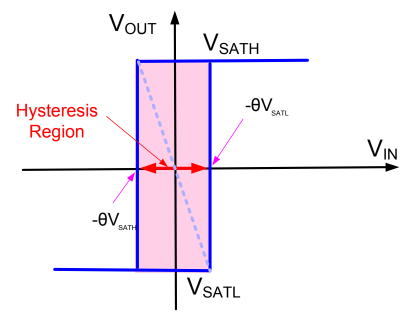
$$\theta = \frac{R_1}{R_1+R_2}$$





$$\theta = \frac{R_1}{R_2}$$

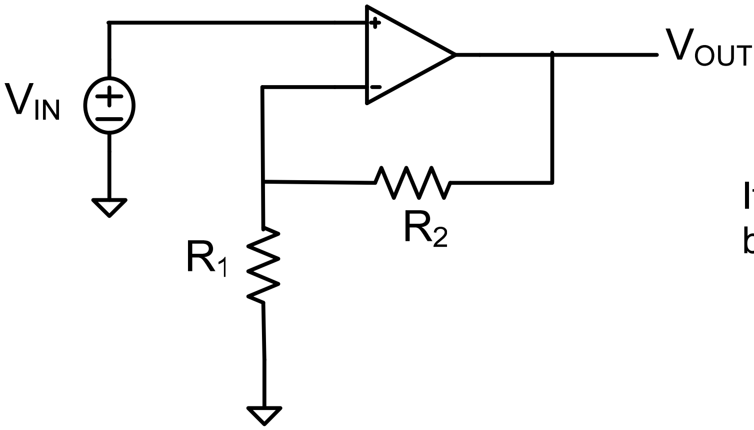
Note this is the basic inverting amplifier with op amp terminals interchanged

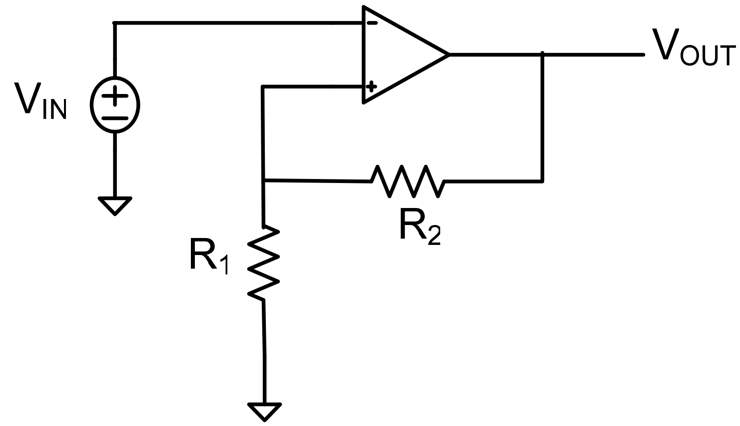


Many other ways to control position and size of hysteresis window

Review from Last Lecture

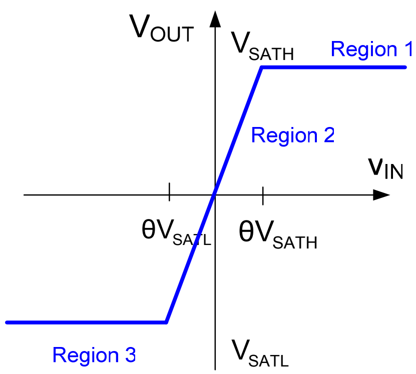
Comparison of basic noninverting amplifier structures

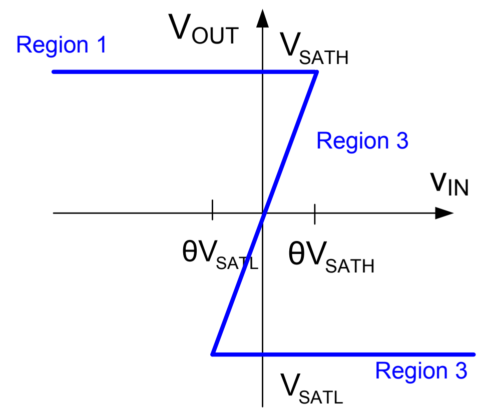




If ideal op amps both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$



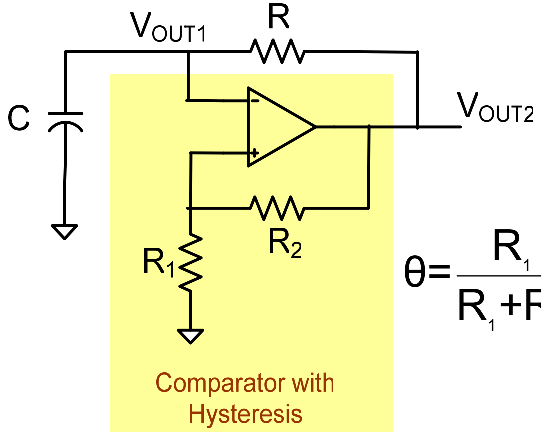


- Serves as an amplifier <u>directly</u>
- Stable
- No hysteresis loop

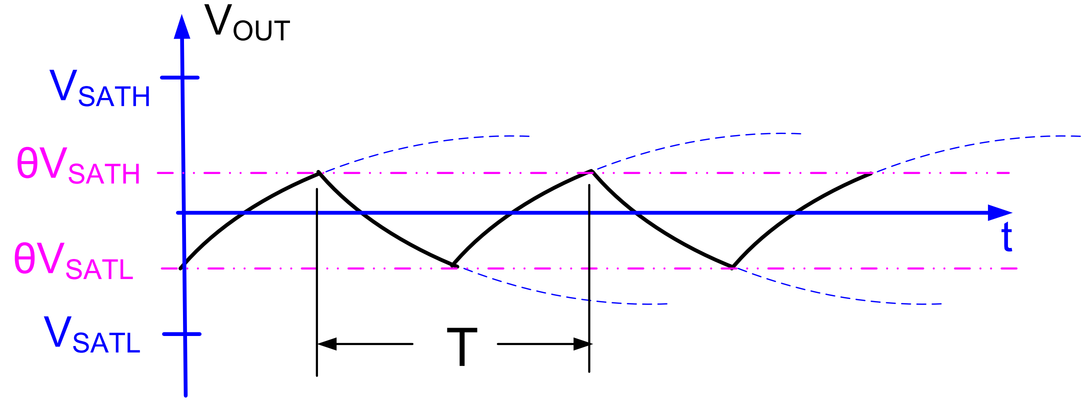
- Not useful as an amplifier <u>directly</u>
- Unstable
- Serves as comparator with hysteresis

Review from Last Lecture

Waveform Generator



$$\theta = \frac{R_1}{R_1 + R_2}$$



$$t_1 = -RC\ln\left(\frac{V_{SATL}(\theta\text{-}1)}{\theta V_{SATH}\text{-}V_{SATL}}\right)$$

this process repeats itself

the rise time and the fall times are identical

the period of the nearly triangular waveform is thus $2t_1$

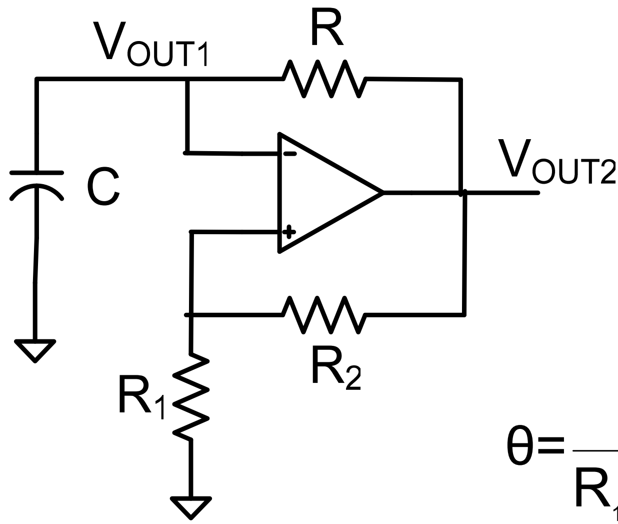
$$T = 2t_1 = -2RC\ln\left(\frac{V_{SATL}(\theta\text{-}1)}{\theta V_{SATH}\text{-}V_{SATL}}\right)$$

$$f = \frac{1}{T} = \frac{1}{2RC}\frac{1}{\ln\left(\frac{\theta V_{SATH}\text{-}V_{SATL}}{V_{SATL}(\theta\text{-}1)}\right)}$$

If V_{SATL}=-V_{SATH} , this simplifies to

$$f = \frac{1}{2RC}\frac{1}{\ln\left(\frac{1+\theta}{1\text{-}\theta}\right)}$$

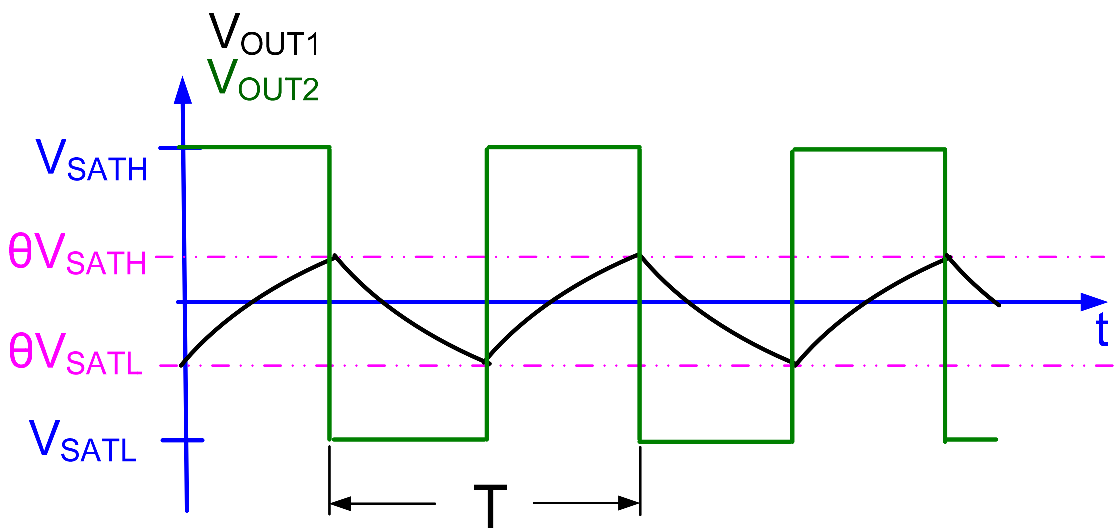
Review from Last Lecture



$$\theta=\frac{R_1}{R_1+R_2}$$

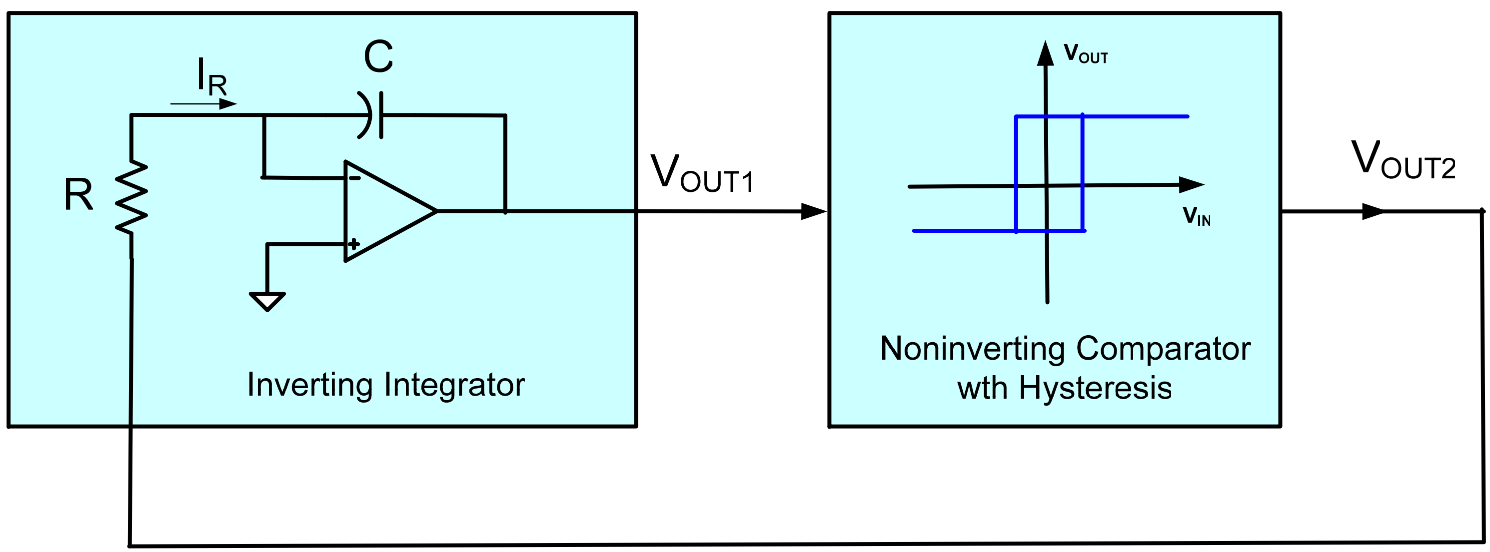
for $V_{SATL}=-V_{SATH}$

$$f=\frac{1}{2RC}\,\frac{1}{\ln\left(\frac{1+\theta}{1-\theta}\right)}$$



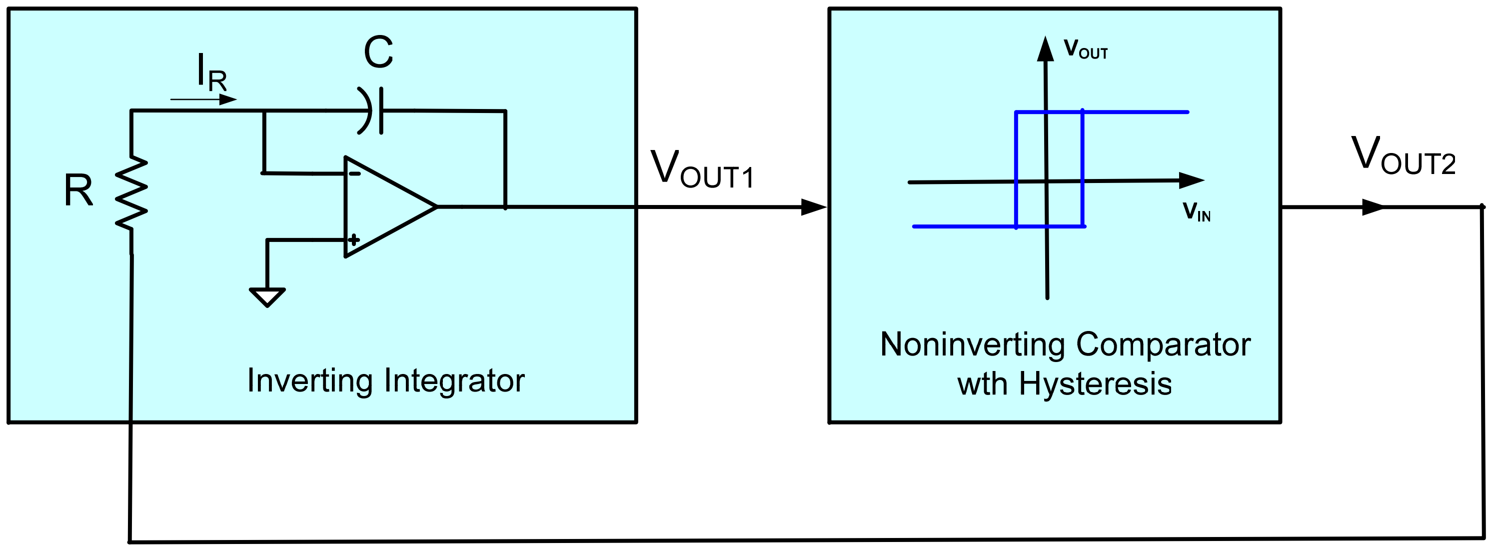
Square and distorted triangular output waveforms
Slope of square wave is determined by SR of Op Amp

Waveform Generator with Linear Triangle Waveform



Goal: Determine how this circuit operates, the output waveforms, and the frequency of the output

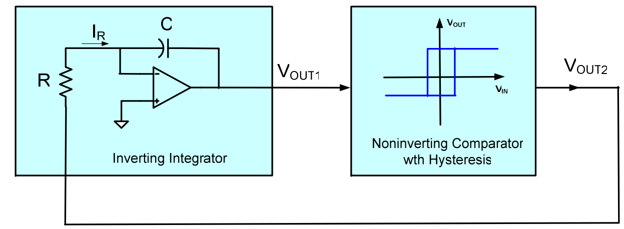
Waveform Generator with Linear Triangle Waveform



Lets first check stability

Since stability is determined by the poles of a linear network, must first assume devices are operating linearly

Waveform Generator with Linear Triangle Waveform



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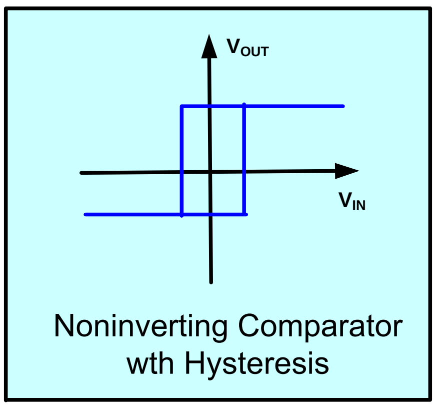
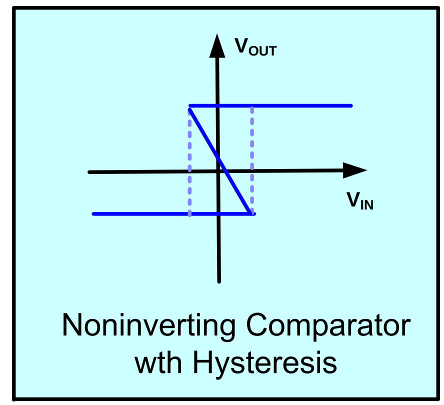


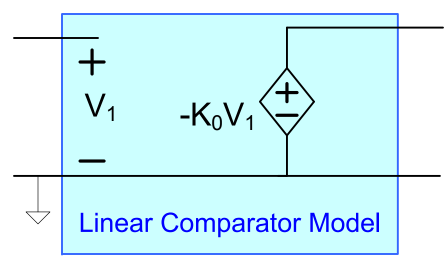
What is the linear model of this comparator?



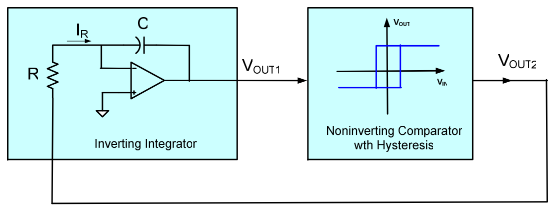
Linear region is area where slope is negative

Recall, in this region,

$$V_{OUT} = -K_0 V_{IN}$$

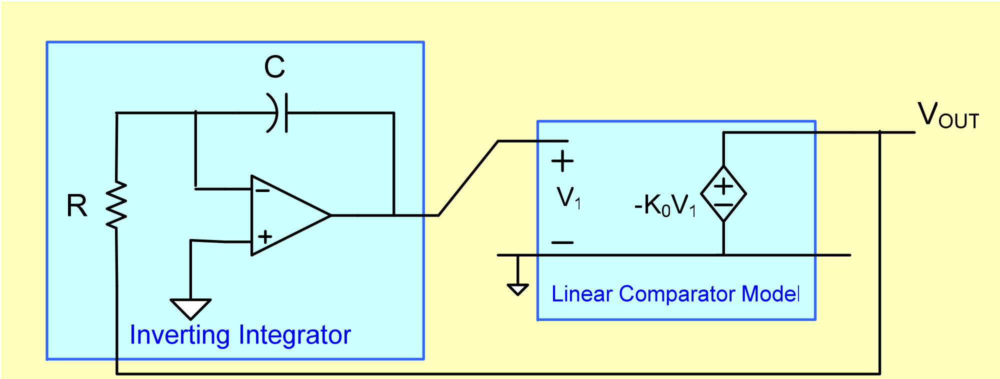


Waveform Generator with Linear Triangle Waveform

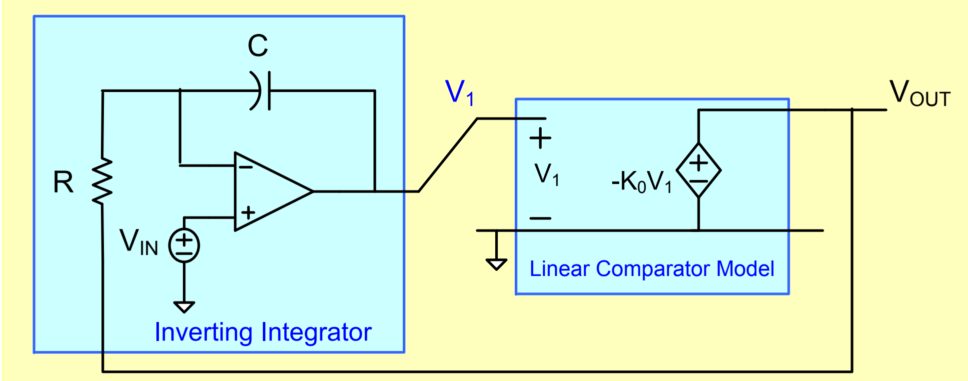


Lets first check stability

Since stability is determined by the poles of a linear network, must first assume devices are operating linearly



Linear circuit model

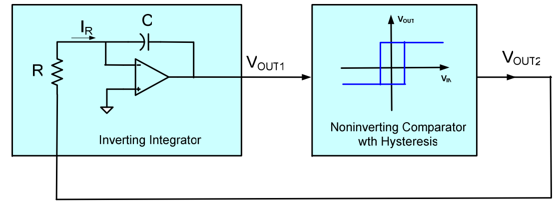


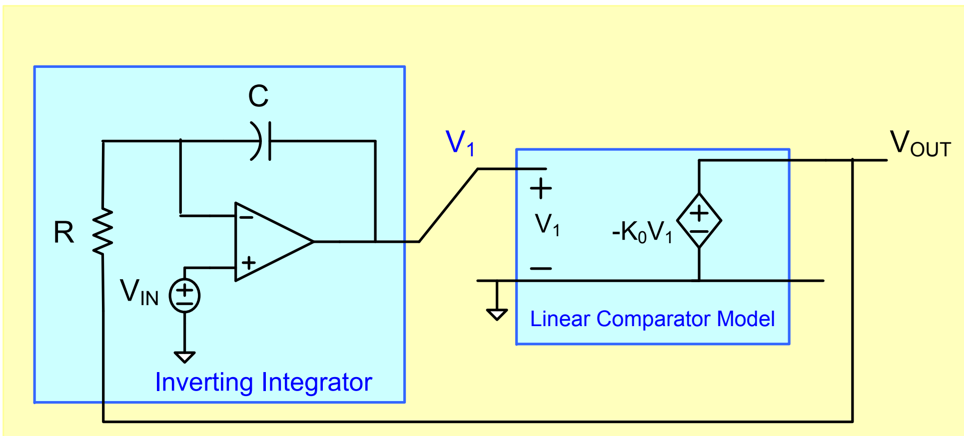
Linear Circuit Model with Excitation

(Recall do not need to provide excitation to find poles but details will be discussed later)

Waveform Generator with Linear Triangle Waveform

Lets first check stability





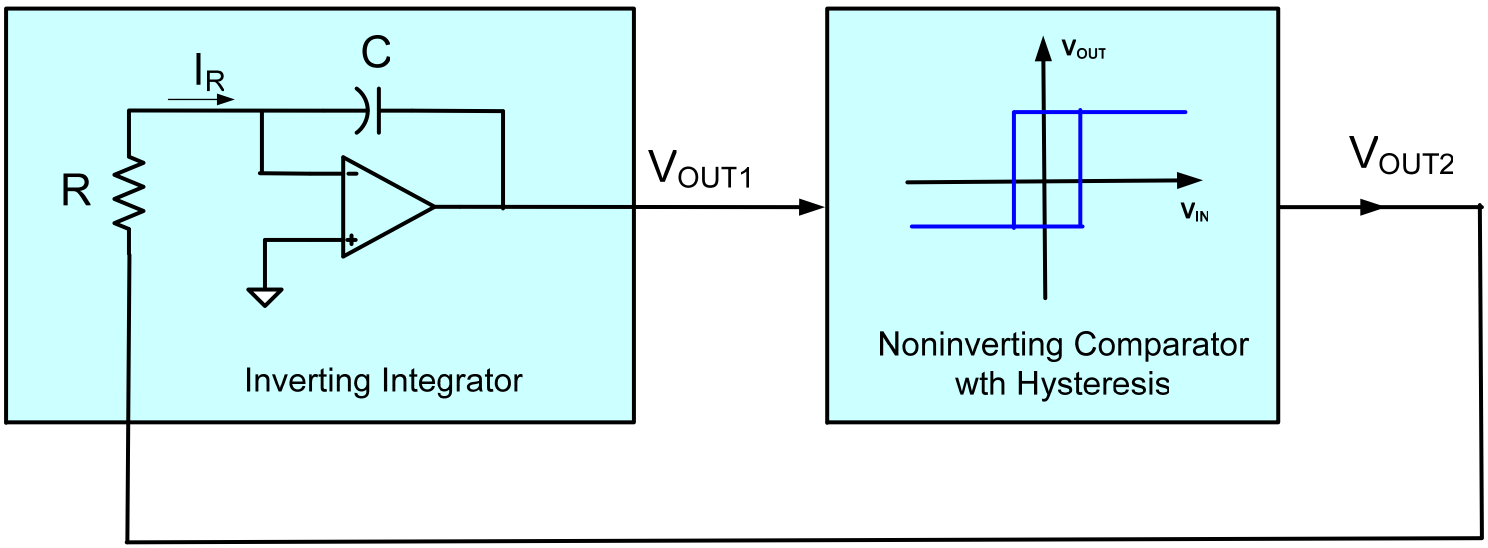
Linear Circuit Model with Excitation

$$\left.\begin{aligned} V_{IN}\left(sC+G\right) &= V_1 sC + V_{OUT}G \\ V_{OUT} &= -K_0 V_1 \end{aligned}\right\} \qquad T(s)=\frac{V_{OUT}}{V_{IN}}=\frac{-K_0\left(s+\dfrac{1}{RC}\right)}{s-\dfrac{K_0}{RC}}$$

Single pole at $s=\dfrac{K_0}{RC}$

$\therefore$ The system is unstable !

Waveform Generator with Linear Triangle Waveform



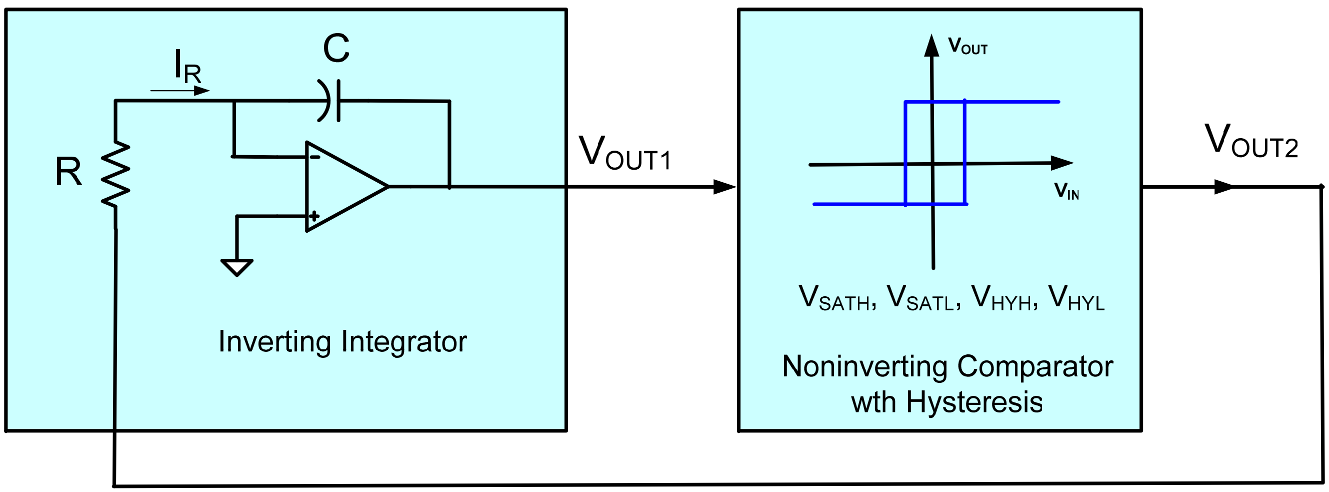
Since the comparator will be in one of two states, the current in the resistor will be constant when $V_{OUT2}=V_{SATH}$ and will be constant when $V_{OUT2}=V_{SATL}$

Analysis strategy: Guess state of the V_{OUT2}, solve circuit, and show where valid

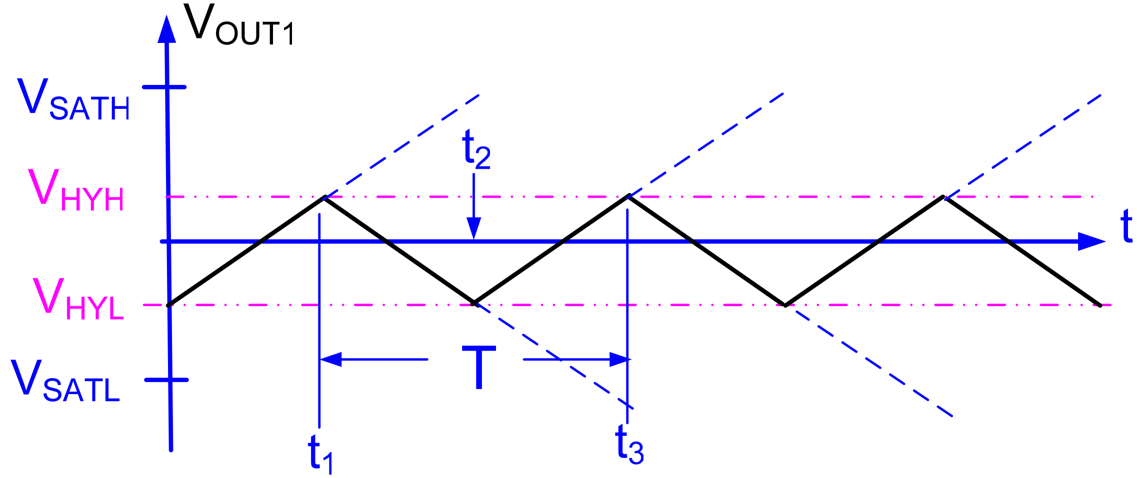
when $V_{OUT2}=V_{SATH}$, I_R will be positive and V_{OUT1} will be decreasing linearly

when $V_{OUT2}=V_{SATH}$, I_R will be positive and V_{OUT1} will be increasing linearly

Waveform Generator with Linear Triangle Waveform

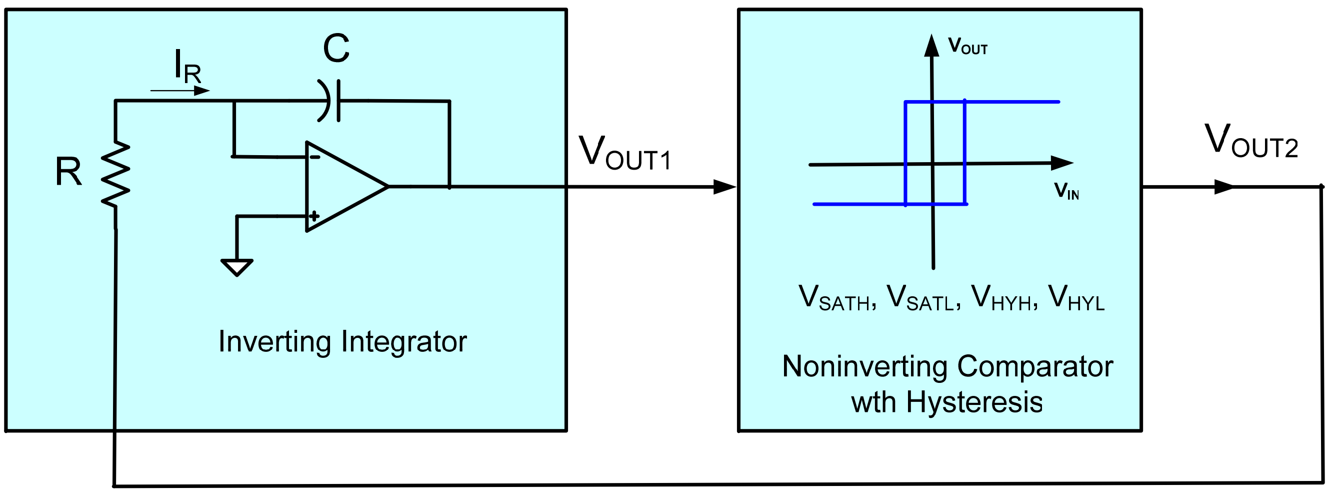


$$V_{SATH} \cong V_{DD}$$
$$V_{SATL} \cong V_{SS}$$



Observe $T = t_3 - t_1 = (t_2 - t_1) + (t_3 - t_2)$

Waveform Generator with Linear Triangle Waveform



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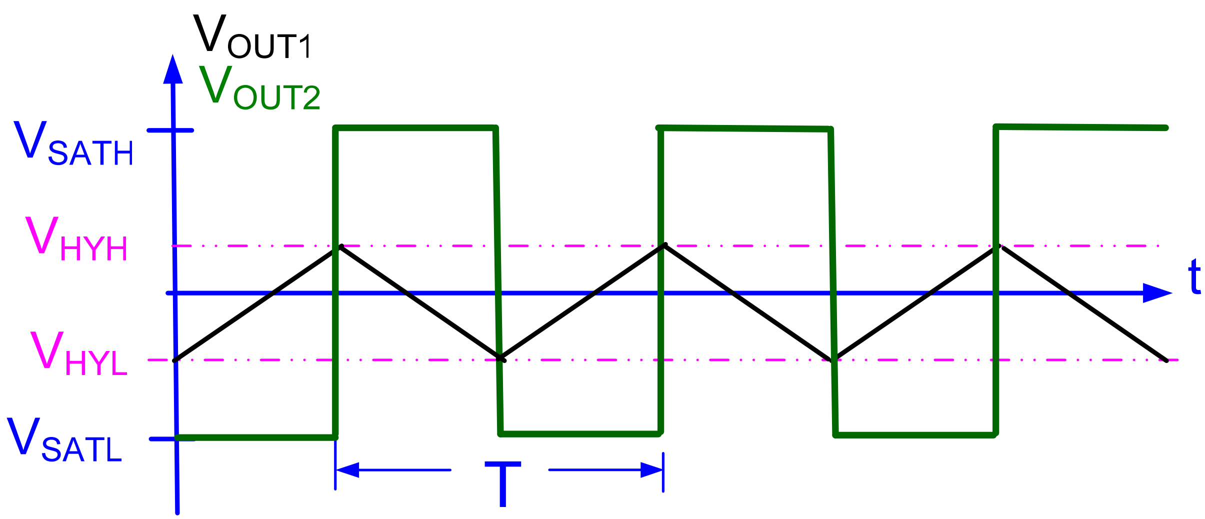
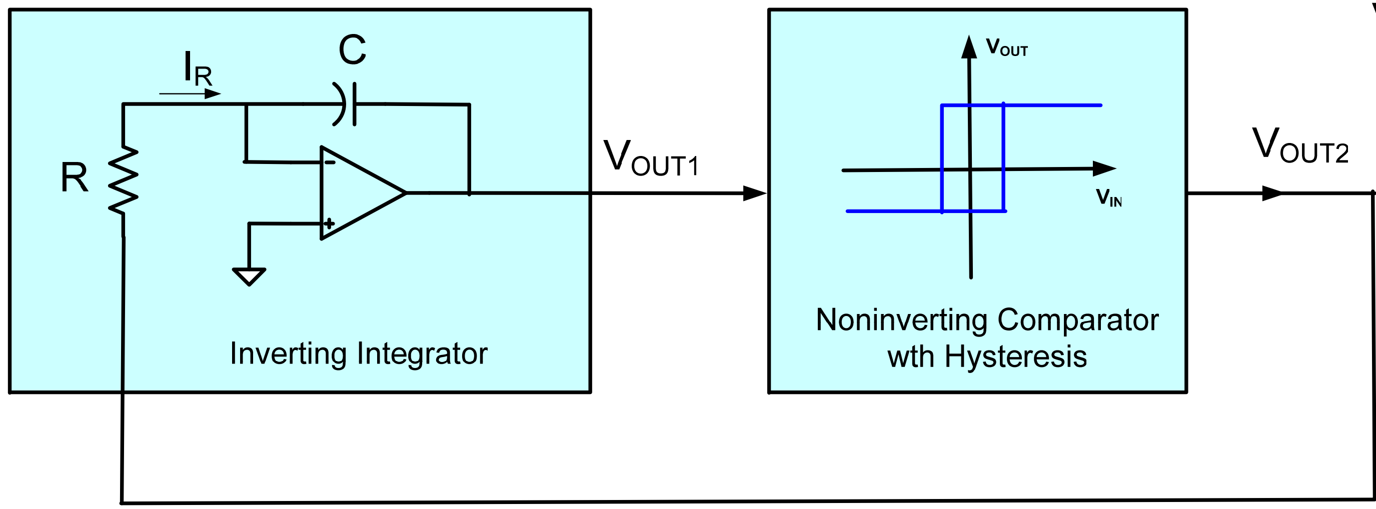


Waveform Generator with Linear Triangle Waveform



$V_{SATH} \cong V_{DD}$

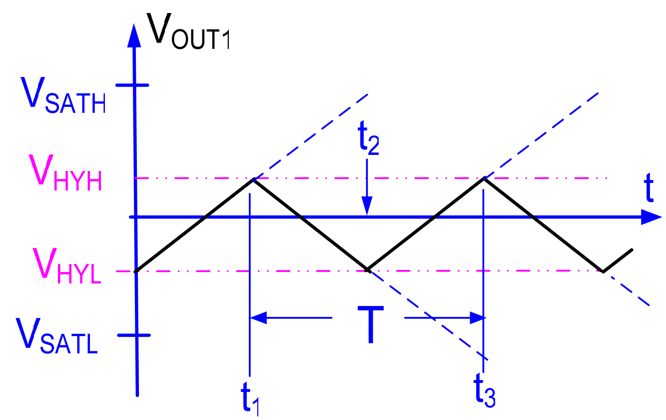
$V_{SATL} \cong V_{SS}$

Guess $V_{OUT2}=V_{SATH}$ will obtain t_2-t_1

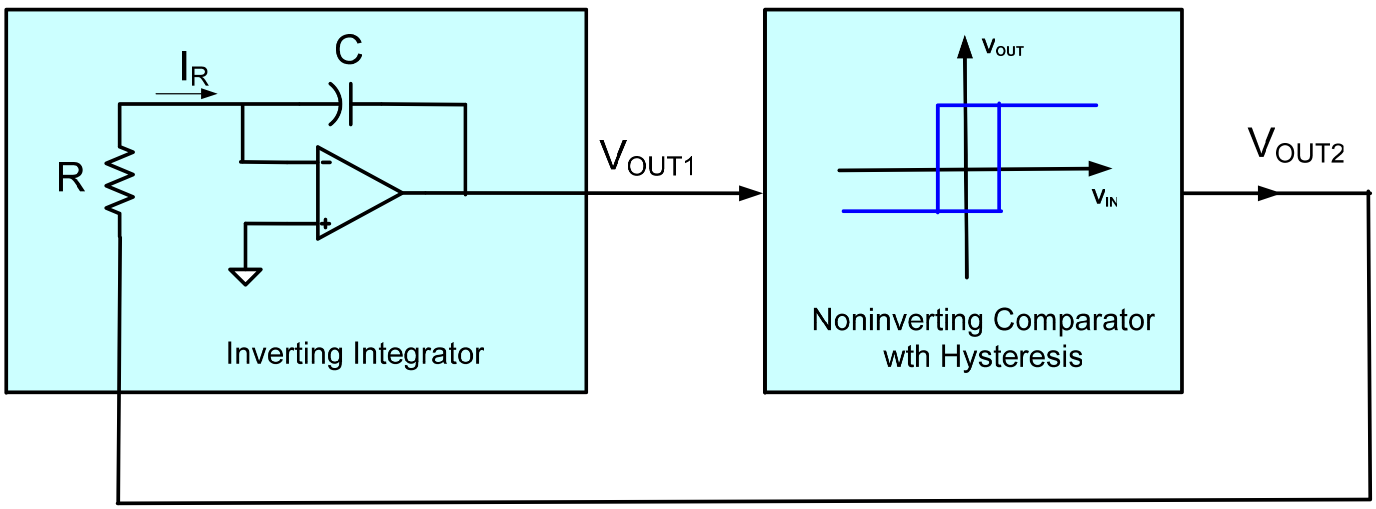
$$V_{OUT1} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{OUT1}(t_1)$$

$$V_{OUT1}(t_1) = V_{HYH}$$

valid for $t_1 < t < t_2$



Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$
$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

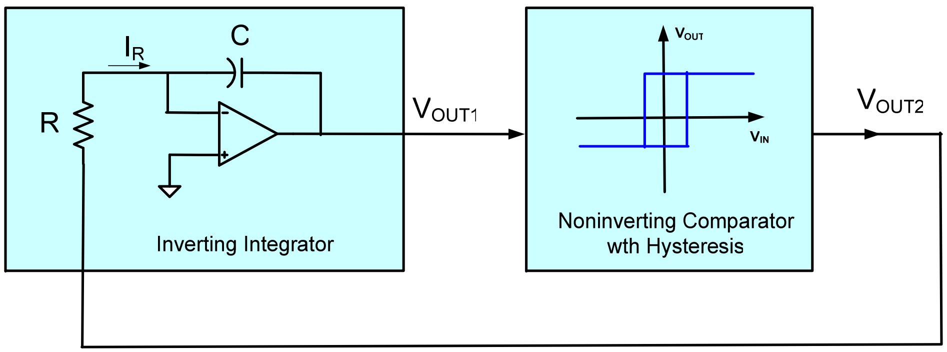
$$V_{OUT1} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{OUT1}(t_1) \qquad V_{OUT1}(t_1) = V_{HYH}$$

at $t=t_2$, V_{OUT1} will become V_{SATL}

Substituting into integral expression for V_{OUT1} we obtain

$$V_{HYL} = -\frac{1}{RC}\int_{t_1}^{t_2} V_{SATH}\,d\tau + V_{HYH}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

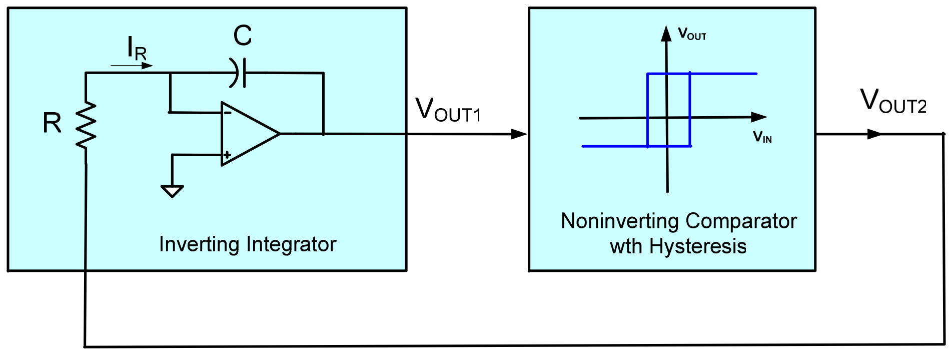
$$V_{HYL} = -\frac{1}{RC}\int_{t_1}^{t_2} V_{SATH}\, d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}\int_{t_1}^{t_2} 1\, d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}\left(\tau\Big|_{t_1}^{t_2}\right) + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}\left(t_2 - t_1\right) + V_{HYH}$$

Waveform Generator with Linear Triangle Waveform



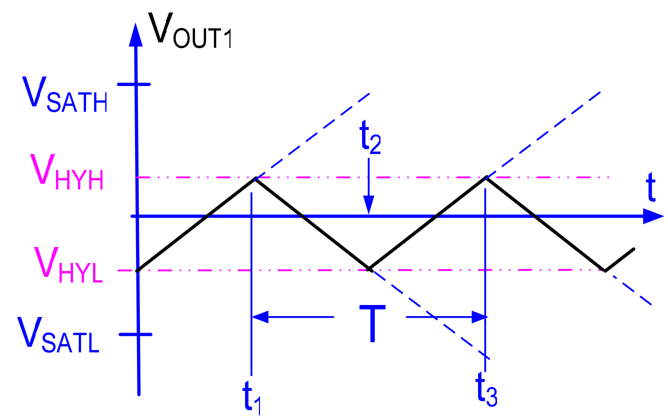
$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

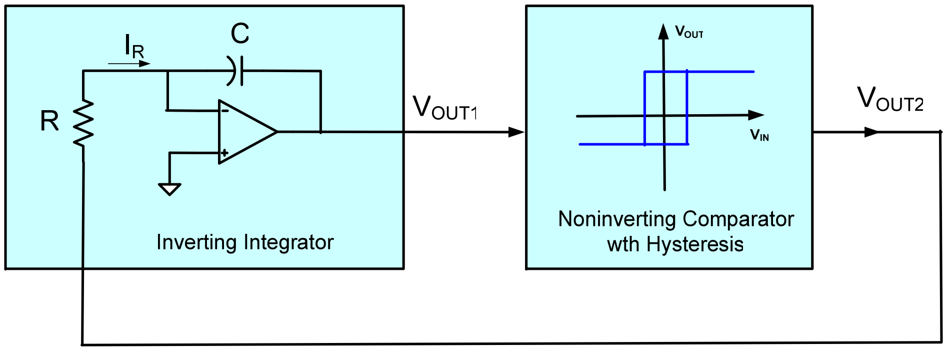
Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}\left(t_2 - t_1\right) + V_{HYH}$$

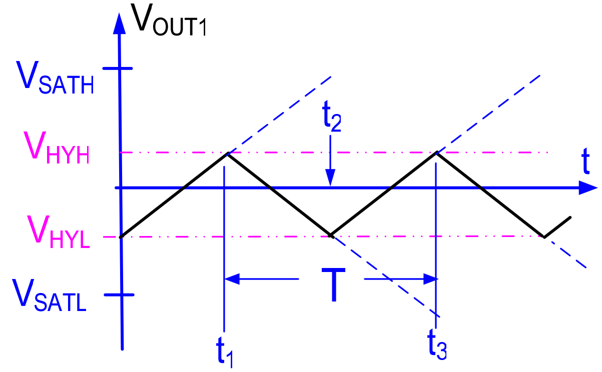
$$t_2 - t_1 = RC\frac{\left(V_{HYH} - V_{HYL}\right)}{V_{SATH}}$$



Waveform Generator with Linear Triangle Waveform



$V_{SATH} \cong V_{DD}$ $\quad V_{SATL} \cong V_{SS}$



Guess $V_{OUT2}=V_{SATL}$ will obtain t_3-t_2

Following the same approach observe (valid for $t_2 < t < t_3$)

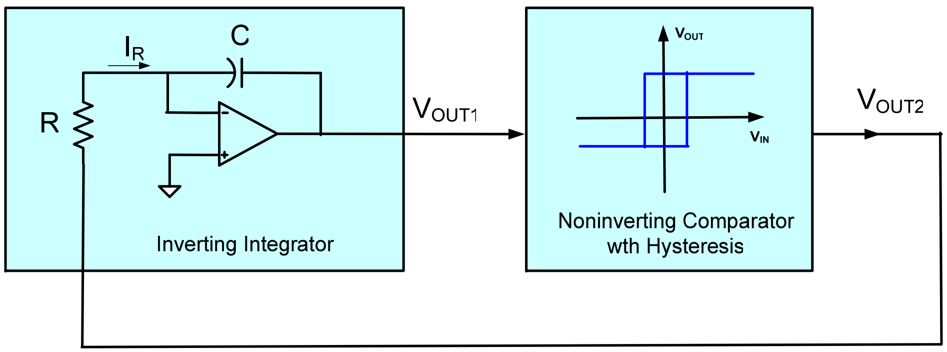
$$V_{OUT1} = -\frac{1}{RC}\int_{t_2}^{t} V_{SATL}\,d\tau + V_{OUT1}(t_2)$$

$$V_{OUT1}(t_2) = V_{HYL}$$

It thus follows that

$$V_{HYH} = -\frac{1}{RC}V_{SATL}(t_3-t_2) + V_{HYL} \qquad t_3-t_2 = RC\frac{(V_{HYL}-V_{HYH})}{V_{SATL}}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

$$T = (t_2 - t_1) + (t_3 - t_2)$$

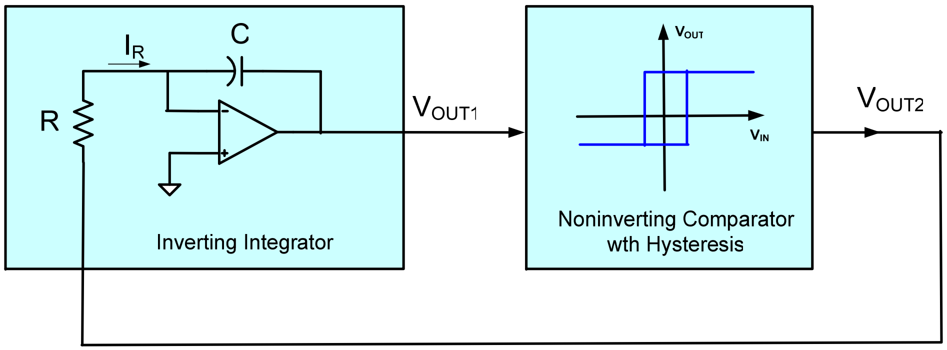
$$t_2 - t_1 = RC\frac{(V_{HYH} - V_{HYL})}{V_{SATH}}$$

$$t_3 - t_2 = RC\frac{(V_{HYL} - V_{HYH})}{V_{SATL}}$$

$$T = RC(V_{HYH} - V_{HYL})\left(\frac{1}{V_{SATH}} - \frac{1}{V_{SATL}}\right)$$

$$f = \frac{1}{t} = \frac{1}{RC}\frac{V_{SATL}V_{SATH}}{(V_{HYH} - V_{HYL})(V_{SATL} - V_{SATH})}$$

Waveform Generator with Linear Triangle Waveform



$$f=\frac{1}{RC}\frac{V_{SATL}V_{SATH}}{(V_{HYH}-V_{HYL})(V_{SATL}-V_{SATH})}$$

If we use the noninverting comparator with hysteresis circuit developed previously and if

If $V_{SATH}=V_{DD}$, $V_{STAL}=V_{SS}=-V_{DD}$ $\theta=\frac{R_1}{R_1+R_2}$

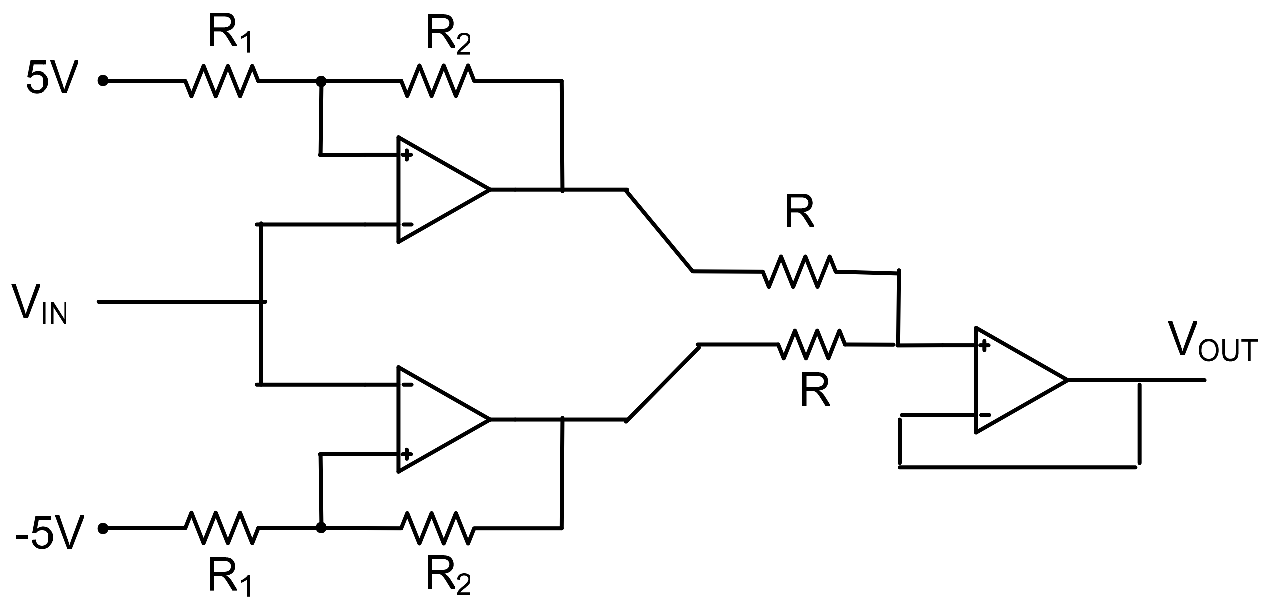
then

$$V_{HYH}=\frac{\theta}{1-\theta}V_{DD} \qquad V_{HYL}=\frac{-\theta}{1-\theta}V_{DD}$$

$$f=\frac{1}{2RC}\frac{1-\theta}{\theta}$$

Example:

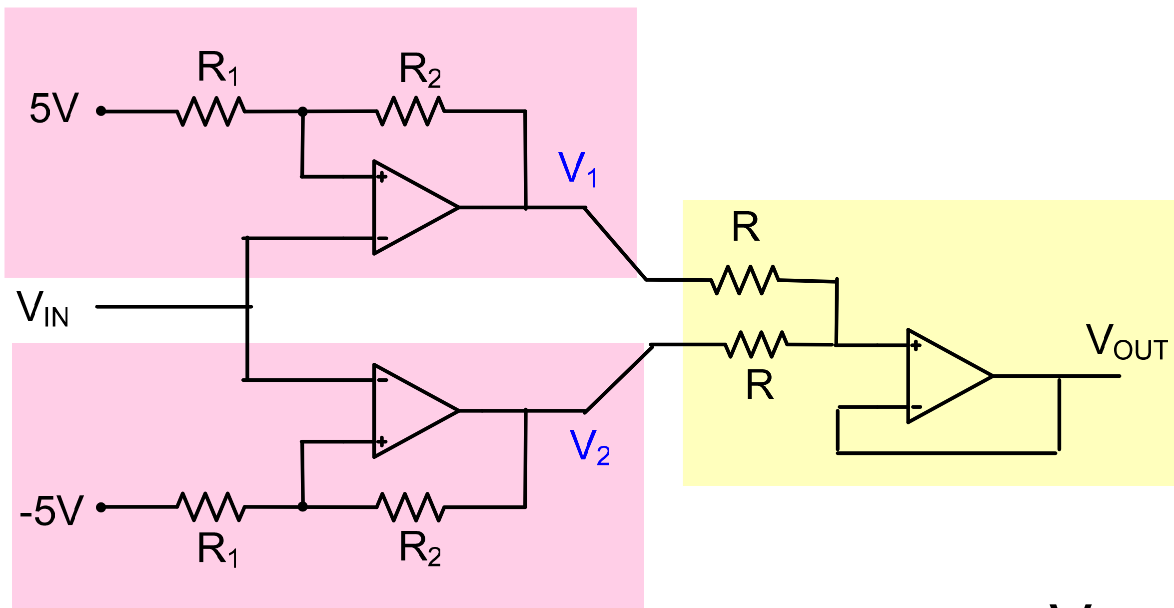
Obtain an expression for and plot the transfer characteristics of the following circuit. Assume R_1=2K, R_2=8K, R=10K, V_{DD}+15V, V_{SS}=-15V



Example:

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume R_1=2K, R_2=8K, R=10K, V_{DD}+15V, V_{SS}=-15V



$$V_{OUT}=\frac{V_1+V_2}{2}$$

$$V_{HYH}=\theta V_{SATH}+(1-\theta)V_R$$

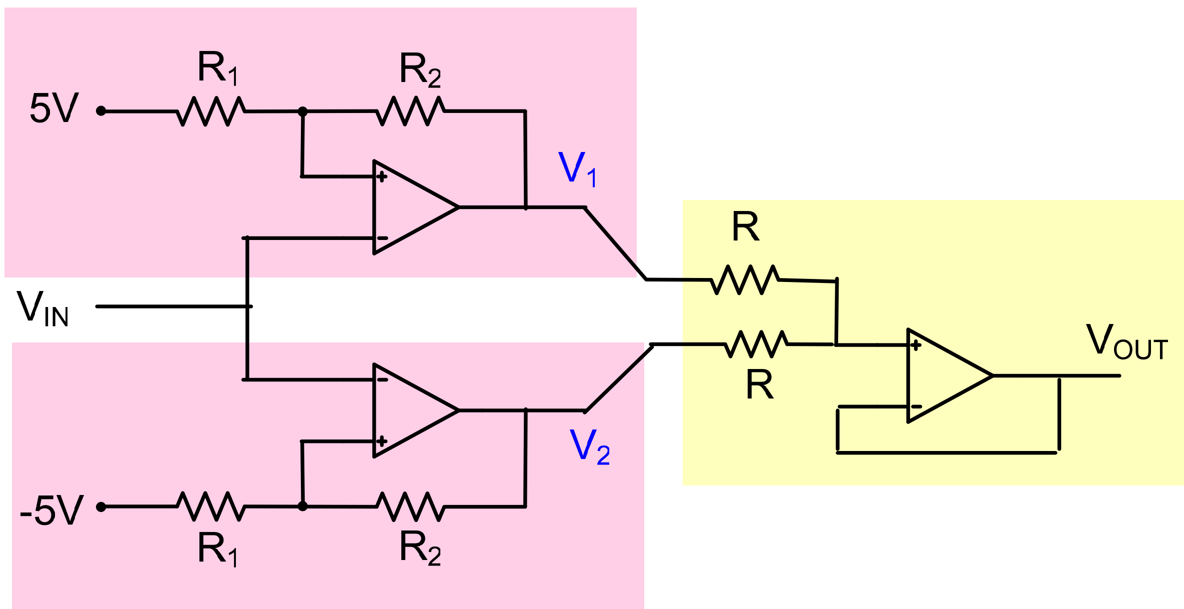
$$V_{HYL}=\theta V_{SATL}+(1-\theta)V_R$$

$$\theta=\frac{R_1}{R_1+R_2}$$

Example:

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume R_1=2K, R_2=8K, R=10K, V_{DD}+15V, V_{SS}=-15V



$$V_{OUT} = \frac{V_1 + V_2}{2}$$

$$\theta = \frac{R_1}{R_1 + R_2} = 0.2$$

Upper Circuit

$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R = 3V+4V=7V$$

$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V+4V=1V$$

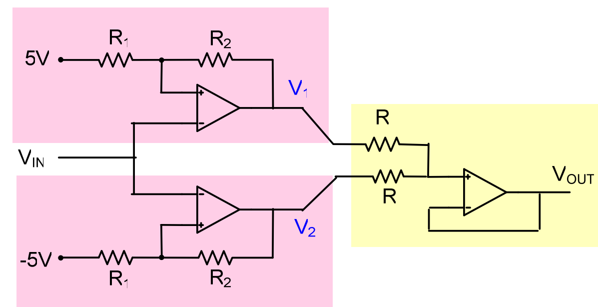
Lower Circuit

$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R = 3V-4V= -1V$$

$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V-4V= -7V$$

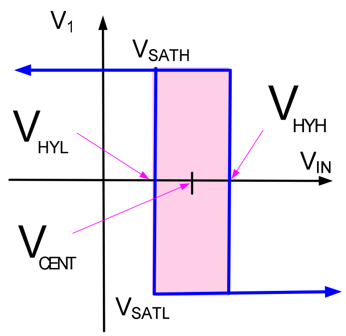
Example:

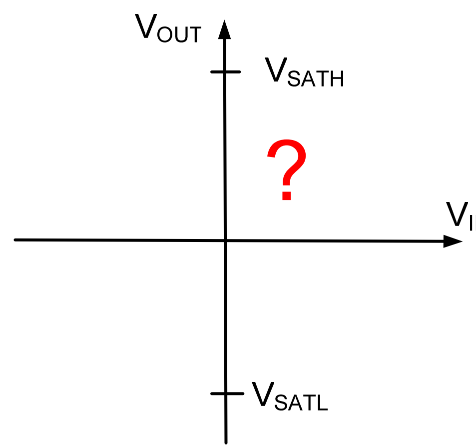
Solution:



$$V_{HYH} = \theta V_{SATH} + (1\text{-}\theta)V_R = 3V+4V=7V$$

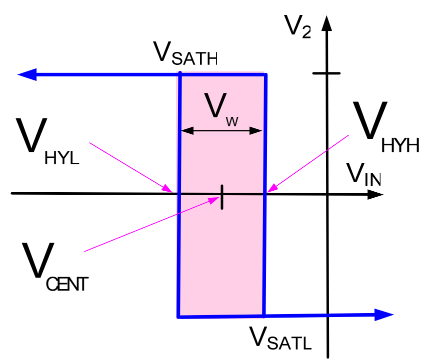
$$V_{HYL} = \theta V_{SATL} + (1\text{-}\theta)V_R = -3V+4V=1V$$





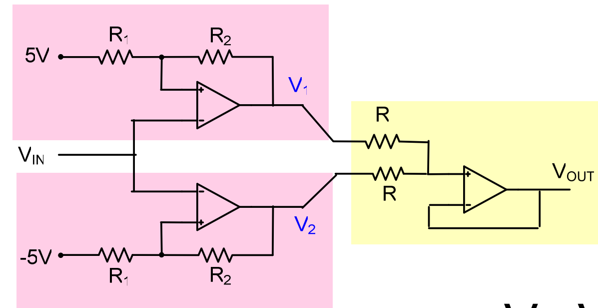
$$V_{HYH} = \theta V_{SATH} + (1\text{-}\theta)V_R = 3V\text{-}4V= -1V$$

$$V_{HYL} = \theta V_{SATL} + (1\text{-}\theta)V_R = -3V\text{-}4V= -7V$$

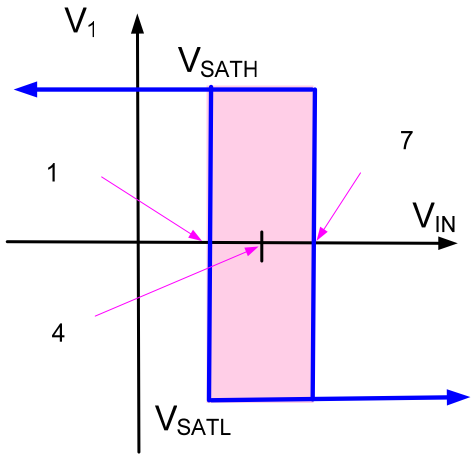


Example:

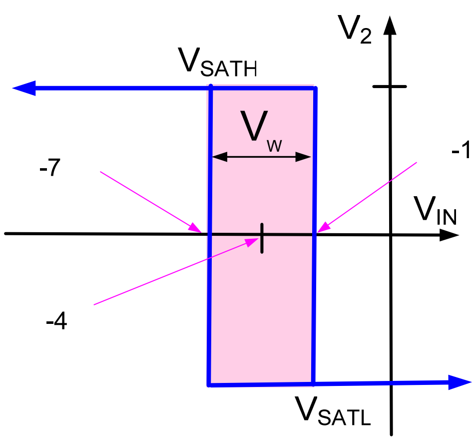
Solution:

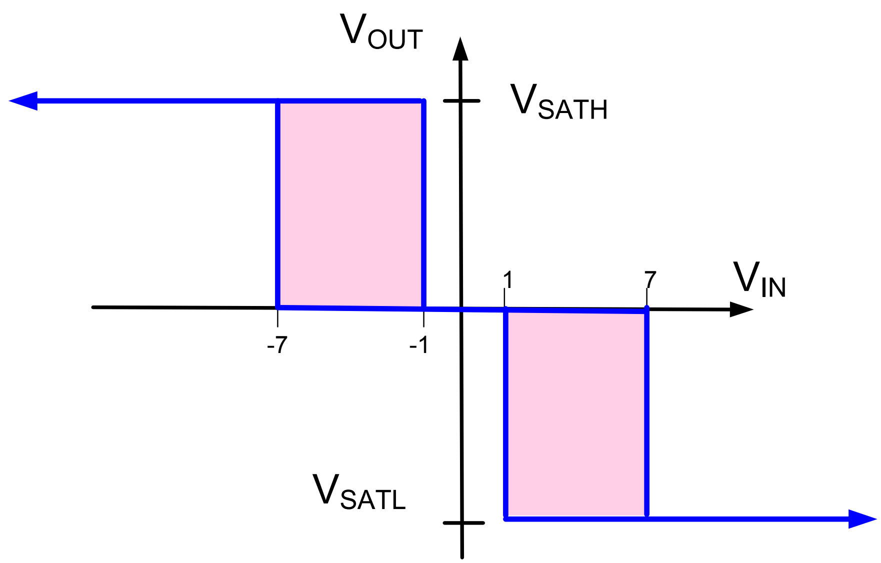


$$V_{OUT} = \frac{V_1 + V_2}{2}$$

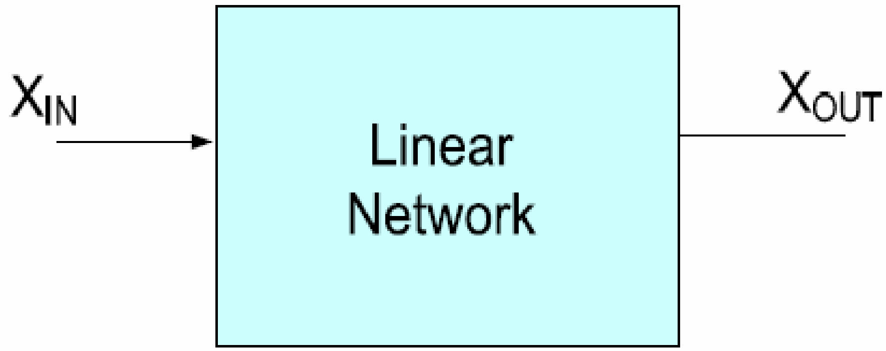


Assuming $V_{SATL} = -V_{SATH}$





Poles of a Network



$$T(s)=\frac{X_{OUT}(s)}{X_{IN}(s)}$$

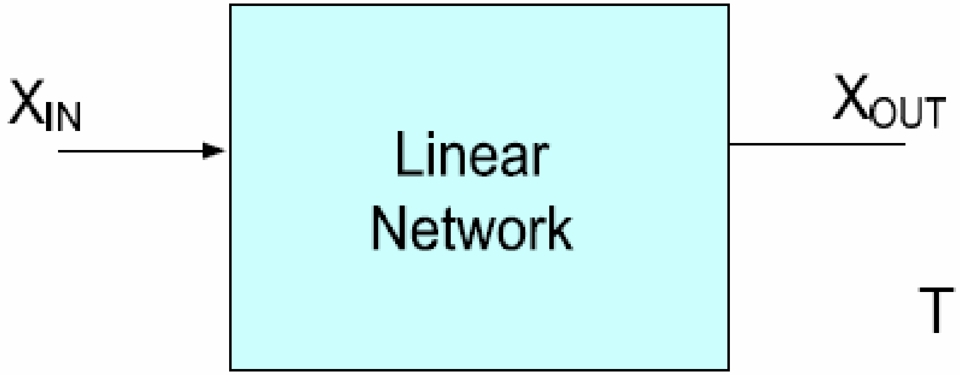
T(s) can be expressed as

$$T(s)=\frac{N(s)}{D(s)}$$

where N(s) and D(s) are polynomials in s

- D(s) is termed the characteristic equation or the characteristic polynomial of the network
- Roots of D(s) are the poles of the network

Poles of a Network



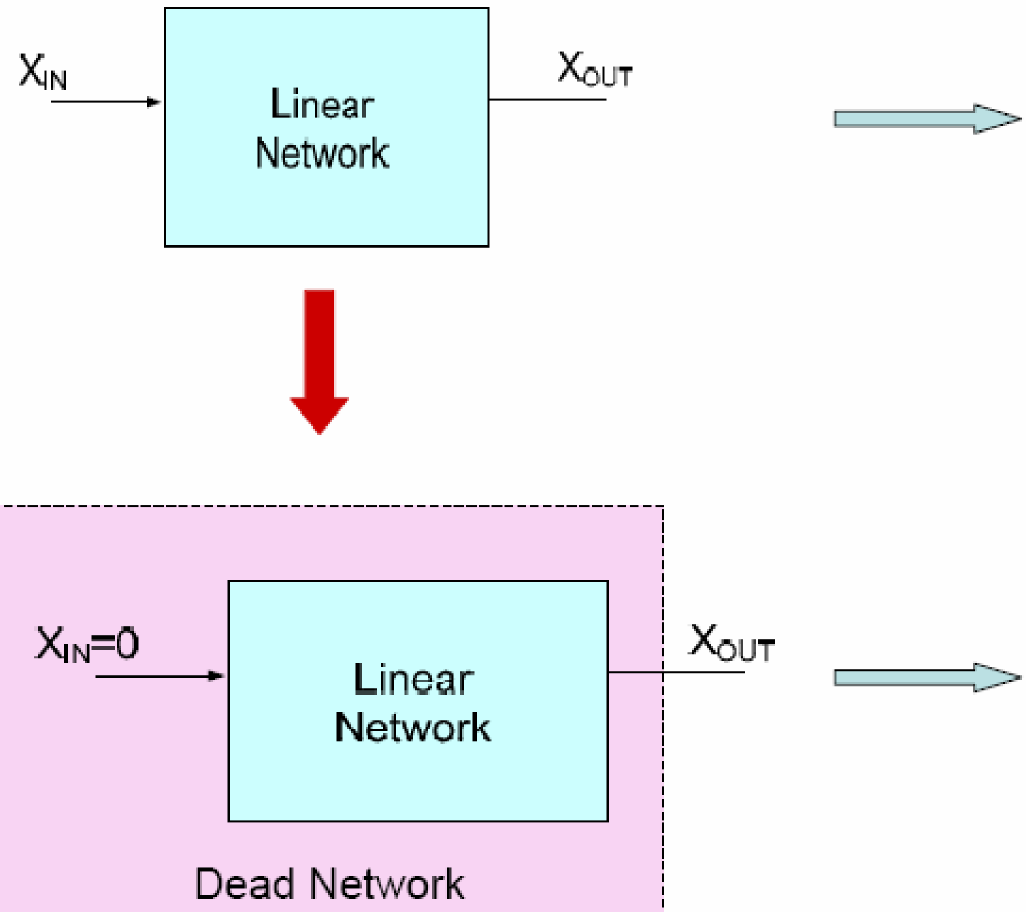
$$T(s)=\frac{N(s)}{D(s)}$$

Theorem: The poles of any transfer function of a linear system are independent of where the excitation is applied and where the response is taken provided the dead networks are the same

Equivalently, the characteristic equation, D(s), is characteristic of a network (or the corresponding dead network) and is independent of where the excitation is applied and where the response is taken.

Poles are inherent and unique characteristics of any linear network.

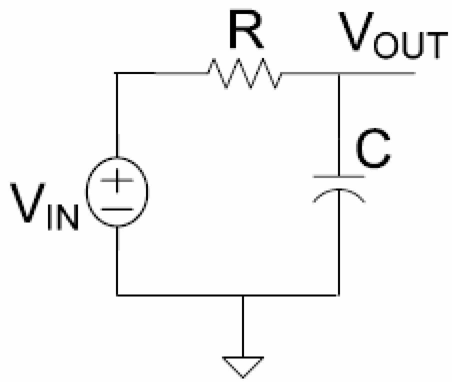
Poles of a Network



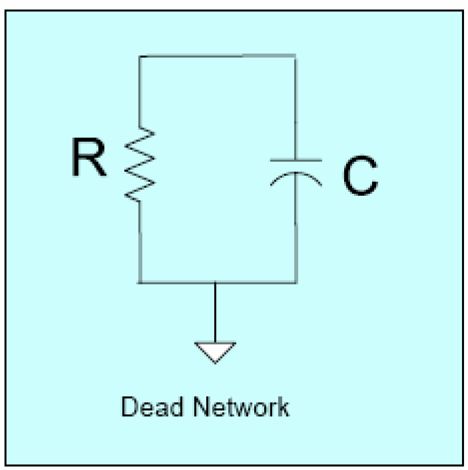
$$T(s)=\frac{N(s)}{D(s)}$$

$$D(s)$$

Poles of Networks – some examples

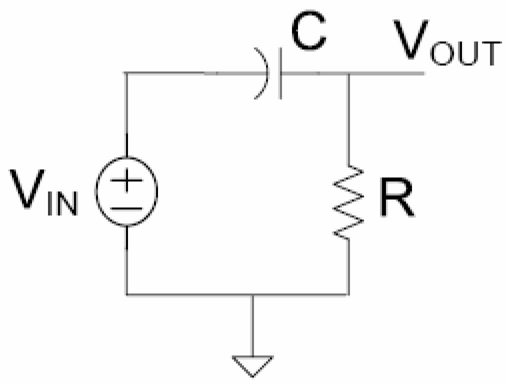


$$T(s)=\frac{V_{OUT}}{V_{IN}}=\frac{1}{1+RCs}$$

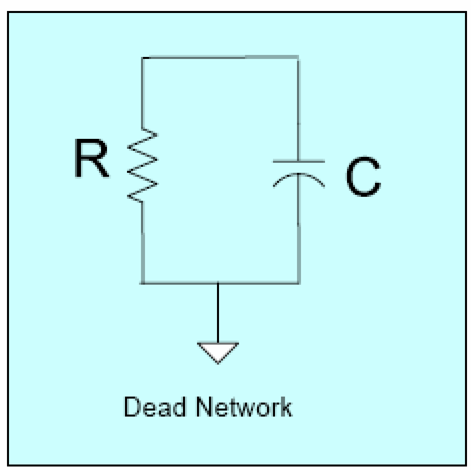


$$D(s)=1+RCs$$

Poles of Networks – some examples

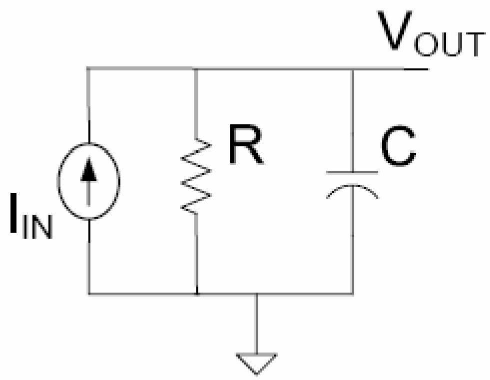


$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{RCs}{1+RCs}$$

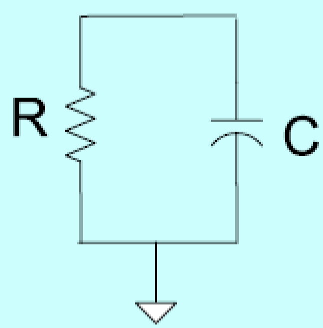


$$D(s) = 1 + RCs$$

Poles of Networks – some examples



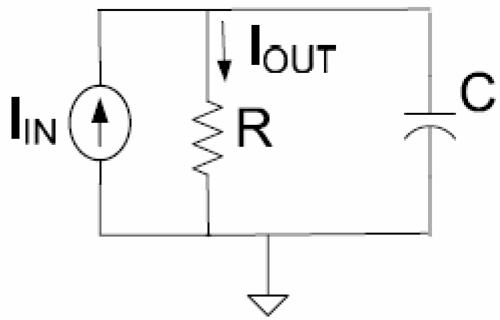
$$T(s) = \frac{V_{OUT}}{I_{IN}} = \frac{R}{1+RCs}$$



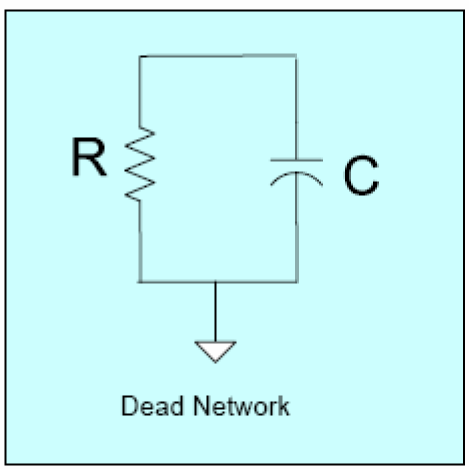
Dead Network

$$D(s) = 1 + RCs$$

Poles of Networks – some examples

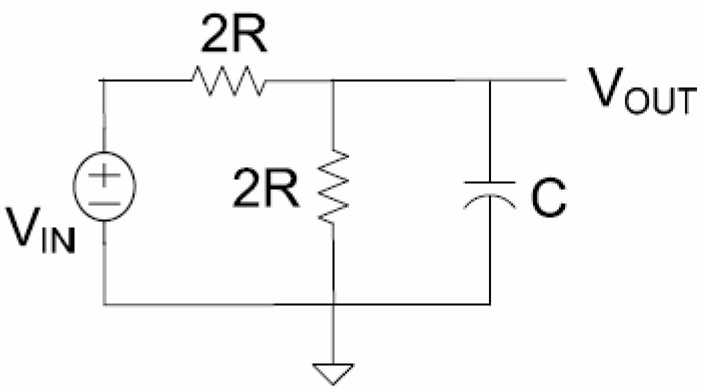


$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{1}{1+RCs}$$

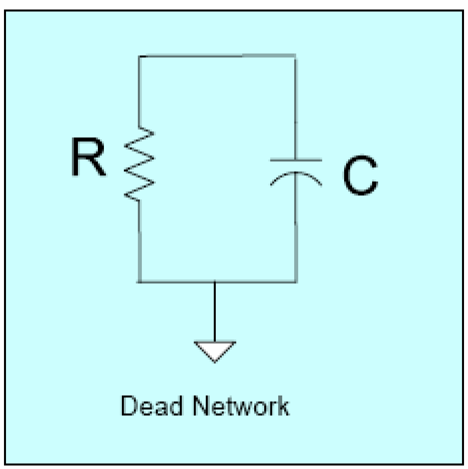


$$D(s)=1+RCs$$

Poles of Networks – some examples

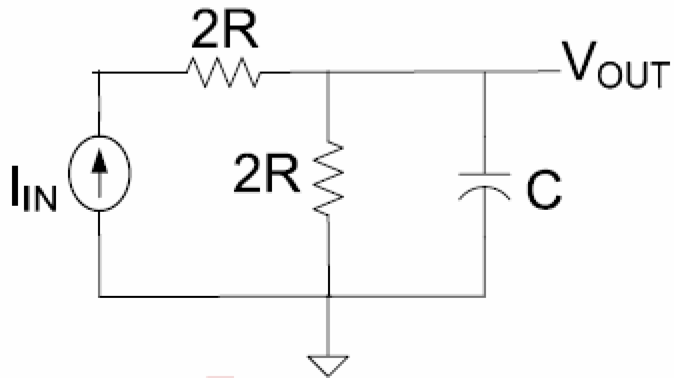


$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{2}}{1+RCs}$$

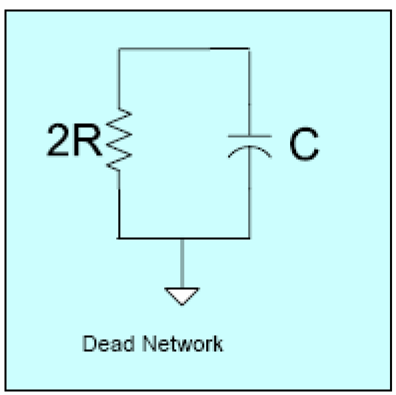


$$D(s) = 1 + RCs$$

Poles of Networks – some examples



$$T(s) = \frac{V_{OUT}}{I_{IN}} = \frac{2R}{1+2RCs}$$



$$D(s) = 1 + 2RCs$$

Note dead network has changed as has D(s) and thus the pole

Strategies for determining poles of networks with no excitations:

1) • Apply any excitation that does not alter the dead network

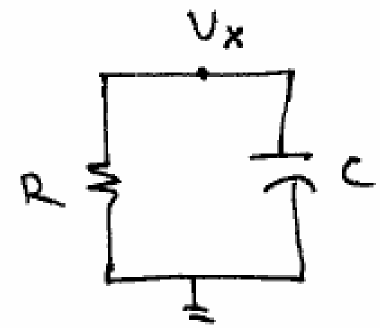
• Obtain transfer function $T(s) = \frac{N(s)}{D(s)}$

• Poles are roots of $D(s)$

• By theorem, they are unique (independent of where excitation is applied or response is taken

2) Develop new strategy that does not require assigning excitation and reduces calculation requirements

Consider the dead network



By KCL

$$V_x\left(sC + \frac{1}{R}\right) = 0$$

$$V_x\,(RCs + 1) = 0$$

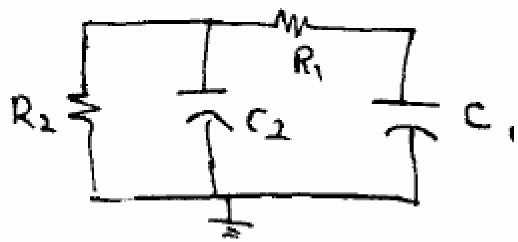
Observe that even though no excitation was applied, the last equation is of the form

$$V_x\, D(s) = 0$$

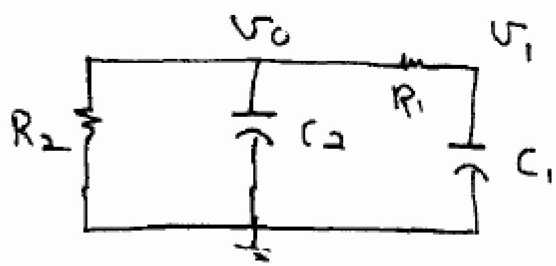
This method for obtaining $D(s)$ is not just a coincidence applicable to this example but rather can be applied to an arbitrary linear network as stated in the following Theorem.

Theorem: The characteristic polynomial $D(s)$ of a system can be obtained by assigning an output variable to the "dead network" and using circuit analysis techniques to obtain an expression that involves only the output variable expressed in the form $X_o F(s) = 0$. When expressed in this form, $F(s)$ when written in polynomial form is the characteristic polynomial of the system. i.e. $D(s) = F(s)$.

Example. Determine the characteristic polynomial for the following circuit



Solution: Assign V_0 to one of the nodes



$\left(G_1 = \frac{1}{R_1}, \; G_2 = \frac{1}{R_2}\right)$

By KCL

$$\left.\begin{aligned} V_0 \left(G_2 + G_1 + sC_2\right) &= V_1 G_1 \\ V_1 \left(G_1 + sC_1\right) &= V_0 G_1 \end{aligned}\right\}$$

Eliminating V_1 between these two equations we obtain

$$V_o\,(G_1 + G_2 + sC_2) = G_1 V_o \frac{G_1}{G_1 + sC_1}$$

or

$$V_o\,(G_1^2 + G_1G_2 + sC_2G_1 + sC_1G_2 + sC_1G_1 + s^2C_1C_2 - G_1^2) = 0$$

or

$$V_o\,(s^2C_1C_2 + s(C_1[G_1+G_2] + C_2G_1) + G_1G_2) = 0$$

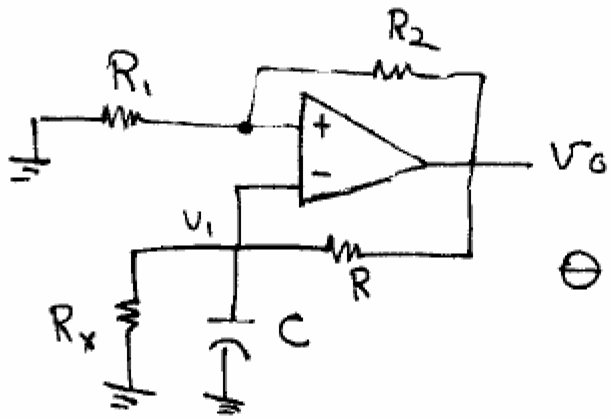
or

$$V_o\left(s^2 + s\left(\frac{[G_1+G_2]}{C_2} + \frac{G_1}{C_1}\right) + \frac{G_1G_2}{C_1C_2}\right) = 0$$

The characteristic polynomial for this circuit is

$$D(s) = s^2 + s\left(\frac{[G_1+G_2]}{C_2} + \frac{G_1}{C_1}\right) + \frac{G_1G_2}{C_1C_2}$$

Consider again the waveform generator. A resistor R_x has been added but if $R_x = \infty$ this becomes the waveform generator considered earlier



$$\theta = \frac{R_1}{R_1 + R_2}$$

Obtain the poles of this circuit. Assume OA is ideal

$$\left.\begin{aligned} V_1 &= \theta V_o \\ V_1 \cdot \left(\frac{1}{R} + \frac{1}{R_x} + sC\right) &= \frac{V_o}{R} \end{aligned}\right\}$$

solving, we obtain

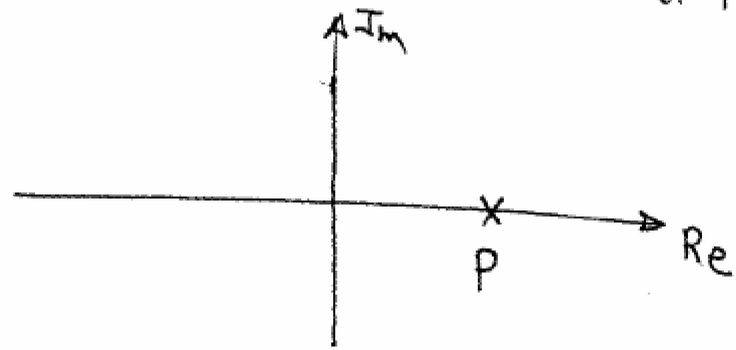
$$V_o\left(s + \left[\frac{1}{RC}\frac{(\theta - 1)}{\theta} + \frac{1}{R_x C}\right]\right)$$

$$\therefore \quad D(s) = s + \frac{1}{RC}\frac{(\theta-1)}{\theta} + \frac{1}{R_x C}$$

If $R_x = \infty$, this circuit has a single pole at

$$p = \frac{1}{RC}\left(\frac{1-\theta}{\theta}\right)$$

Since $0 < \theta < 1$, this circuit has a single pole on the positive real axis and is unstable



Determine the minimum value of R_x that will maintain instability in the previous circuit.

$$D(s) = s + \frac{1}{RC}\left(\frac{\theta - 1}{\theta}\right) + \frac{1}{R_x C}$$

$\therefore$ pole at

$$P = \frac{1-\theta}{\theta}\left(\frac{1}{RC}\right) - \frac{1}{R_x C}$$

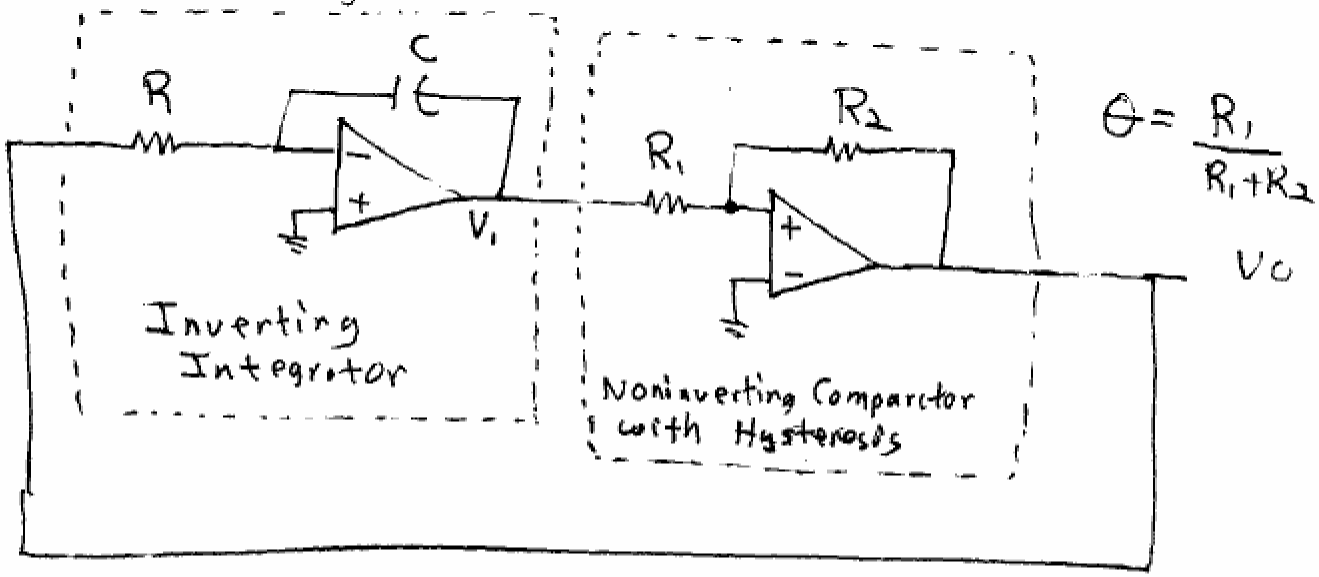
setting this equal to 0 (when it leaves RHP) obtain

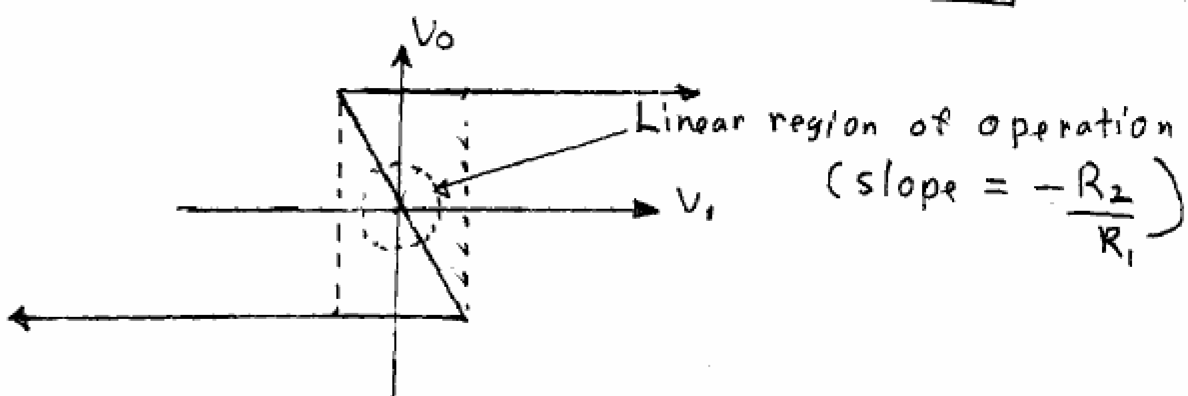
$$\frac{1}{C R_{x\,min}} = \frac{1-\theta}{\theta}\left(\frac{1}{RC}\right)$$

or

$$R_{x\,min} = R\left(\frac{\theta}{1-\theta}\right)$$

Where are the poles of the following waveform generator?





Transfer Characteristics of Comparator

Must use linear mode of operation to find poles of the circuit

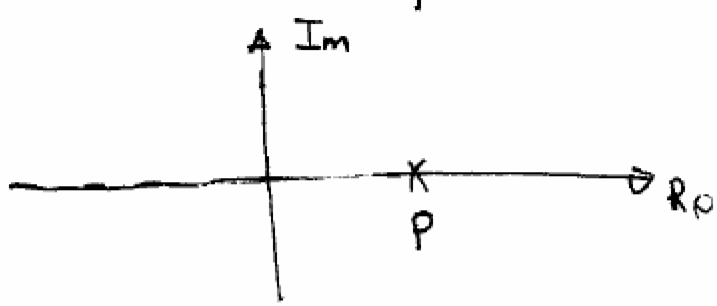
$$\left.\begin{aligned} V_o &= -\frac{R_2}{R_1} V_1 \\ V_1 &= -\frac{1}{RCs} V_o \end{aligned}\right\}$$

solving, obtain

$$V_o \left(s - \frac{R_2}{R_1} \frac{1}{RC} \right) = 0$$

$\therefore$ single pole at $p = \left(\frac{R_2}{R_1}\right) \frac{1}{RC}$

Note pole in RHP on positive real axis



Stability and Waveform Generation

- Waveform generators provide an output with no excitation
- Waveform circuits are circuits that, when operated in quiescent linear condition, have one or more poles in the right half-plane
- Will now investigate the pole locations of waveform generators
  - Conditions for oscillation
  - Triangle/Square/Sinusoidal Oscillations

End of Lecture 23